

A MATLAB FE Library for the Simulation and Inversion of EM Problems

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MT 2D & 3D

Mount Tarawera

Reconstruction of seven conductive bodies and two conductive plates embedded in a homogeneous half-space from the synthetic data set COPROD_2S1.

Preliminary inversion of a GNS MT data set acquired in the vicinity of mount Tarawera, New Zealand [1].

Mesh: vertical section

Surface mesh

Discretization of the Tarawera region using Gmsh.

Pole-dipole & Schlumberger measurement

DC 2D, 2.5D & 3D

Pseudosection showing $\rho_{a,obs}$

2.5D simulation and inversion

Inversion of a field data set over an assumed 2D conductive anomaly in Freiberg.

READY TO USE

2D & 3D FEM BUILDING BLOCKS [8]

Matrix assemblies	Div-Grad	Mass matrix Jacobian	Mass	Curl-Curl
Element types and orders	Lagrange 1. to n-th order [F2]	Raviart-Thomas 1. order [F2]	Nédélec 1. & 2. order [F2]	
Boundary conditions	Robin	Dirichlet inh. & hom.	Neumann inh. & hom.	
Source types	Point	Line	Face	Volume

INVERSION BUILDING BLOCKS

- Scalable iterative solver [9] for H^1 -regularized normal equations [10]
- Gauss-Newton approach
- Explicit Jacobian assembly
- Line search (Armijo) [11]

2D & 3D MESHING BY Gmsh [3]

- Triangle & tetrahedra meshing
- Boolean & refinement operators
- Handy .geo format

3D VISUALIZATION BY ParaView [5]

- Export of .vtk & .xdmf

SOLVER

- MUMPS [6] parallel direct
- HSL_MI20 [7] AMG

TEM

- Spatial discretization available (CSEM)
- To do:
 - Measurement operator
 - Time integration by rational best approximation [12]
 - Assembly of Jacobian

UNDER DEVELOPMENT

CSEM

Sensitivity distribution (real part) for an exciting line source (~40Hz) on top of a layered half-space and a B_x receiver coil at a depth of about 220m.

- Line sources (wires / loops supported)
- Data types: E, H
- 3D Inversion prototype available
- Nédélec elements, hom. Dirichlet BC, total field
- Adaptable to HEM applications

IP

- Complex-valued conductivities
- 2D, 2.5D, 3D
- 3D Inversion prototype available

Phase and resistivity distribution on Fossa volcano crater (Vulcano, Italy). Preliminary results from stacked 2.5D inversions using two crossing profiles from a 3D data set comprising 14 profiles and 19 frequencies.

FUTURE WORK

OTHER TO-DO'S

- Complete documentation
- Expansion of CI test environment
- Enhance set up of inversion parameters
- Data visualization
- GUI interface
- Adaptive mesh refinement, secondary field, total variation

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