Including geological orientation information into geophysical inversions with unstructured tetrahedral meshes

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Summary

A gradient-based roughness operator that has the capability of including geological orientation information such as strike and dip angles into minimum-structure inversions using unstructured tetrahedral meshes is proposed. In contrast to the majority of the gradient-based methods that consider a cell in a package with its neighbours to form these types of roughness operators, our proposed method calculates the roughness operators between the two adjacent cells. Hence, the proposed method is able to construct models with sharper boundaries for the scenarios in which the regularization function is measured by an ℓ_1 norm compared to the methods that consider a cell in a package with its neighbours.

Keywords: Geological orientation information, Roughness operators, Unstructured tetrahedral meshes

Introduction

Roughness operators were introduced into geophysical inversion— minimum-structure or Occam's style of inversion— to reduce the non-uniqueness of the inverse problem (Constable et al., 1987) and to enable one to incorporate a priori information into the inversion framework to obtain more plausible models (e.g., Li & Oldenburg, 2000). Incorporating a priori information such as structural orientation information (strike, and dip angles) into the inversion, particularly for survey methods with limited depth resolution such as gravity, magnetics, electric, and electromagnetic methods such as magnetotellurics (MT), is important.

Designing roughness operators that allow one to incorporate geological orientation information into inversions using unstructured tetrahedral meshes is not as straightforward as for inversions using structured meshes due to the complex geometry of the unstructured meshes (Lelièvre & Farquharson, 2013). Perhaps the most simple and robust method of forming the roughness operators for unstructured tetrahedral meshes is the one that calculates the physical property differences across the internal mesh faces (Günther et al., 2006). However, this method or the methods proposed by Usui (2015) and Özyildirim et al. (2017) are not able to incorporate geological orientation information such as strike, dip, and tilt angles into the inversion framework.

A few methods have been proposed that enable one to incorporate geological orientation information into the inversions using unstructured tetrahedral meshes. The majority of these methods consider a cell in a package with its neighbors to form the roughness operators. For example, Lelièvre & Farquharson (2013) consider a cell in a package with its neighbours that share a common edge/face, Key (2016) consider a cell in a package with its neighbours that share a common node, and Jordi et al. (2018) go beyond the nearest neighbours of each inversion cell using the correlation function to form the roughness operators. Although these methods are able to incorporate geological information into the inversion framework successfully, they are not able to construct models with sharp boundaries for the scenarios in which the regularization function is measured by an ℓ_1 norm due to the package issue.

To address this package issue, we adapt and extend the method proposed by Günther et al. (2006), called xyz-Günther, to form the roughness operators that allow the inclusion of geological information into the inversion framework and to construct piecewise-constant, blocky models for the scenarios that the regularization function is measured using an ℓ_1 norm. Ekblom's measure (Ekblom, 1973) is adopted to measure the regularization function due to its being numerically well-behaved. The iteratively reweighted least squares (IRLS) method (e.g., Farquharson & Oldenburg, 1998) is utilized to minimize the inverse problem. In the following, the minimum-structure inverse problem is briefly described, then the xyz-Günther method and its capability on synthetic gravity data of a dipping prism are investigated.

Minimum-structure inversion

The objective function that we design to do a minimum-structure inversion consists of a data-misfit term, ϕ_d, and a regularization term, ϕ_m, (e.g., Constable et al., 1987; Smith & Booker, 1988),

$$\Phi(\mathbf{m}) = \phi_d(\mathbf{m}) + \beta\phi_m(\mathbf{m}), \tag{1}$$

where the model vector $\mathbf{m}$ contains the physical property values of the inversion cells. The trade-off parameter, β, controls the relative contribution of the data-misfit and the regularization terms in the objective function. The data-misfit term measures the difference between the observed noisy data, d_i^{obs}, and predicted data, d_i^{pred}, which is scaled by the standard deviation of the noise, σ_i:

$$\phi_d(\mathbf{m}) = \sum_{i=1}^{N_d} \Big(\frac{d_i^{obs} - d_i^{pred}(\mathbf{m})}{\sigma_i}\Big)^2, \tag{2}$$

where N_d is the number of data points. The regularization term consists of a roughness term, ϕ_r, and a smallness term, ϕ_s, (Li & Oldenburg, 1998),

$$\phi_m(\mathbf{m}) = \alpha_r\, \phi_r(\mathbf{m}) + \alpha_s\, \phi_s(\mathbf{m}), \tag{3}$$

where the roughness term measures the amount and type of model structure and the smallness term measures the difference between the constructed model and the reference model, m_{ref},

$$\begin{aligned}\phi_m(\mathbf{m}) =&\alpha_r \int_v \Big(W(\mathbf{r})\,\frac{\partial m}{\partial r}\Big)^p\, dv\, + \\ &\alpha_s \int_v \Big(W(\mathbf{r})\,(m - m_{ref})\Big)^p dv,\end{aligned} \tag{4}$$

where p represents an ℓ_p-norm measure and $W(\mathbf{r})$ is the distance/depth/sensitivity weighting function. Potential data such as gravity and magnetic data have limited depth resolution (Li & Oldenburg, 1998), hence, weighting functions are applied to the roughness and smallness terms to counteract the natural decay of the kernels with depth and consequently prevent the construction of the features in the model near the surface (Lelièvre & Oldenburg, 2009).

The roughness term $\dfrac{\partial m}{\partial r}$ in Eq. 4 follows the approach of Günther et al. (2006) and measures the physical property differences between two adjacent cells (and is not a full gradient of the model). This roughness term in this form is not able to incorporate geological orientation information into the inversion framework. To address this problem, we adopt and extend this method, called xyz-Günther, such that one be able to incorporate geological information into the inversion framework.

xyz-Günther method

Our proposed method calculates the directional derivatives, $\dfrac{\partial m}{\partial r}\hat{r}$, instead of the derivatives, $\dfrac{\partial m}{\partial r}$, of the physical properties between the two adjacent cells,

$$\begin{aligned}\alpha_r\left(\frac{\partial m}{\partial r}\hat{r}\right) = \left(\frac{\partial m}{\partial r}\frac{1}{r}\right)\Big\{&\alpha_x\,(x_2 - x_1)\,\hat{i}\, + \\ &\alpha_y\,(y_2 - y_1)\,\hat{j} + \alpha_z\,(z_2 - z_1)\,\hat{k}\Big\},\end{aligned} \tag{5}$$

where r is the distance between the centres of the two adjacent cells, and $\hat{r}$ is the unit vector directed between the centres of the two adjacent cells. The x_1, y_1, z_1 and x_2, y_2, z_2 are the coordinates of the centres of the two adjacent cells.

To obtain the roughness operators in the geology coordinate system, we follow Li & Oldenburg (2000) and apply the rotation matrix $\mathbf{R}$ which contains orientation information of geological structure on the roughness operators calculated in the Cartesian coordinate system (i.e., Eq.5):

$$\Big(\frac{\partial m}{\partial x'}, \frac{\partial m}{\partial y'}, \frac{\partial m}{\partial z'}\Big)^T = \mathbf{R}\Big(\frac{\partial m}{\partial x}, \frac{\partial m}{\partial y}, \frac{\partial m}{\partial z}\Big)^T. \tag{6}$$

To construct models with sharp boundaries, the regularization function can be measured by non-ℓ_2-norm measures (e.g., Farquharson & Oldenburg, 1998; Farquharson, 2008). Ekbloms's measure (Ekblom, 1973), which is a perturbed version of an ℓ_p norm, is adopted to measure the regularization function due to its being numerically well-behaved:

$$\rho(x_i) = (x_i^2 + \varepsilon^2)^{p/2}, \tag{7}$$

where x_i are elements of the vector that is supposed to be minimized, and ε is a small number. As the ε gets small, this measure approaches an ℓ_p norm. The iteratively reweighted least squares (IRLS) method (e.g., Farquharson & Oldenburg, 1998) is utilized to minimize our objective function.

Examples

To evaluate the capability and the performance of the proposed xyz-Günther method, the vertical component of the surface gravity data of a dipping prism (Fig. 1) is inverted. The strike, dip, and tilt angles of the dipping prism are (0°, 45°, 0°). The linear trend approach (Lelièvre & Farquharson, 2013) was implemented and applied to the same model to investigate the package issue. To invert the data, Gaussian noise with zero mean and standard deviation of 1% of the maximum absolute value of the data was added to the synthetic data generated using Okabe's method (Okabe, 1979).

Fig. 2 illustrates the constructed density models of the dipping prism for the scenarios that the regularization function is measured by an ℓ_2 norm. The left, middle, and right panels are, respectively, associated with the density models constructed by the original method of Günther et al. (2006), the xyz-Günther method, and the linear trend approach (Lelièvre & Farquharson, 2013) by assigning smoothness weights (α'_x, α'_y, α'_z)=(1.0, 1.0, 10^4) everywhere in the inversion domain. The constructed density models using the xyz-Günther method (panels b & e) and the linear trend approach (panels c & f) have a better representation of the dipping prism than the model constructed using the Günther et al. (2006) method (panels a & d) due to incorporating geological information into the inversion framework.

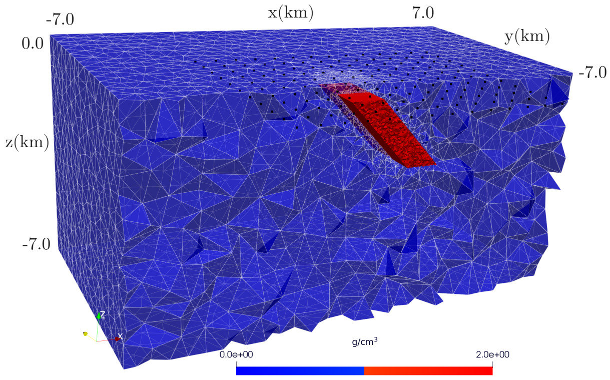

Figure 1: A 3D view of the dipping prism model.

In contrast to the fuzzy and smeared-out density models constructed by an ℓ_2 norm (Fig. 2), the density models constructed by an ℓ_1 norm (Fig. 3) are piecewise-constant, and blocky with sharper boundaries. The left, middle, and right panels are associated, respectively, with the density models constructed by the Günther et al. (2006) method, the xyz-Günther method, and the linear trend approach (Lelièvre & Farquharson, 2013) by assigning (α'_x, α'_y, α'_z)= (1.0, 1.0, 10^3) everywhere in the inversion domain. Although the constructed density models using the xyz-Günther method (panels b & e) and the linear trend approach (panels c & f) have a better representation of the dipping prism extension compared to the model constructed using the Günther et al. (2006) method (panels a & d), the constructed density models using the Günther et al. (2006) method still demonstrate that the subsurface structure is not vertical and has a dip. The density models constructed using the Günther et al. (2006) method (panels a & d) suggest that the diagonal matrices that Farquharson (2008) introduces into the inversion framework using structured meshes to impose a dip on the models constructed by an ℓ_1 norm are not needed for the inversions using unstructured tetrahedral meshes.

The constructed density models using the Günther et al. (2006) method and the xyz-Günther method have a sharper boundary than the model constructed using the linear trend approach due to the package issue. The package issue that the linear trend approach suffers from gets more severe for the methods that consider a larger number of neighbour cells of the inversion cell to form the gradient operators (e.g., Key, 2016; Jordi et al., 2018).

The top panels in Figs. 2 & 3 demonstrate the density models constructed by applying the depth weighting function outside the roughness operators, i.e., the proper location, and the bottom panels of Figs. 2 & 3 demonstrate the density models constructed for the scenarios that the depth weighting function is applied inside the roughness operators. These examples demonstrate that, although applying the depth weighting function inside or outside the roughness operators measured by an ℓ_2 norm does not affect the density models, applying this function inside or outside the roughness operators measured by an ℓ_1 norm does affect the constructed density models. The density models constructed by applying the depth weighting function inside the roughness operators have a trend and are not as sharp as the density models constructed by applying the depth weighting function outside the roughness operators.

Conclusion

The synthetic gravity examples demonstrate that the proposed method xyz-Günther is able to incorporate geological orientation information effectively into the inversion procedure. This method is also able to construct more piecewise-constant models with sharper boundaries compared to the models constructed using methods that consider each cell in a package with its neighbours if the regularization term is measured by an ℓ_1 norm instead of an ℓ_2 norm. The examples also show that applying the depth weighting function inside or outside of the roughness operators measured by an ℓ_1 norm does affect the constructed models. Future work will involve implementing the xyz-Günther method to electromagnetic inversion.

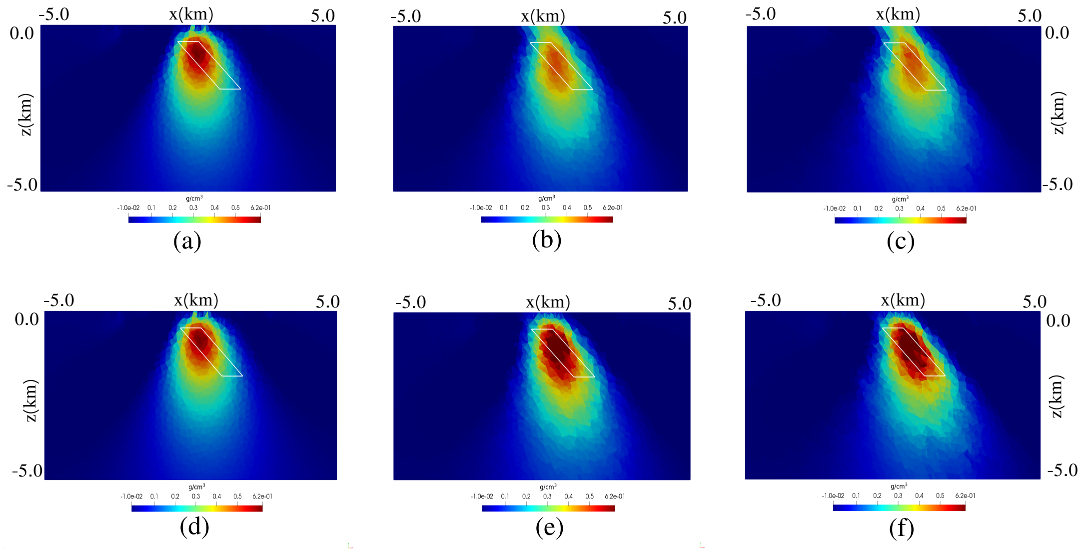

Figure 2: Vertical sections along the x-axis of the density models constructed using an ℓ_2 norm. The left, middle, and right panels correspond to the 3D density models constructed using the Günther et al. (2006) method, the xyz-Günther method, and the linear trend approach (Lelièvre & Farquharson, 2013), respectively. The top and bottom panels correspond to the scenarios that the depth weighting function is applied outside and inside of the gradient operators, respectively. The white solid line indicates the location of the true model.

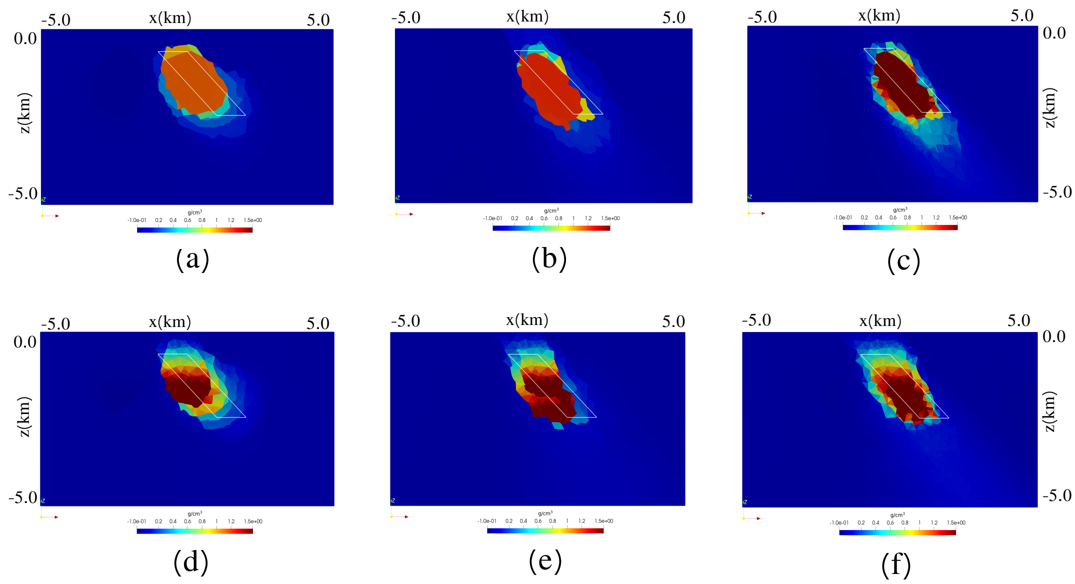

Figure 3: Vertical sections along the x axis of the density models constructed using an ℓ_1 norm. The left, middle, and right panels correspond to the 3D density models constructed using the Günther et al. (2006) method, the xyz-Günther method, and the linear trend approach (Lelièvre & Farquharson, 2013), respectively. The top and bottom panels correspond to the scenarios that the depth weighting function is applied outside and inside of the gradient operators, respectively. The white solid line indicates the location of the true model.

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